4.5 Prove Triangles Congruent by SAS and HL

Before You used the SSS Congruence Postulate.

Now You will use sides and angles to prove congruence.

Why? So you can show triangles are congruent, as in Ex. 33.

G.CO.10 Prove theorems about triangles.

Included Angle - The angle included between two sides.

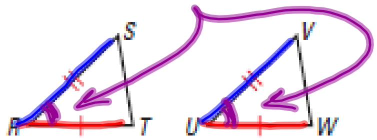
POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

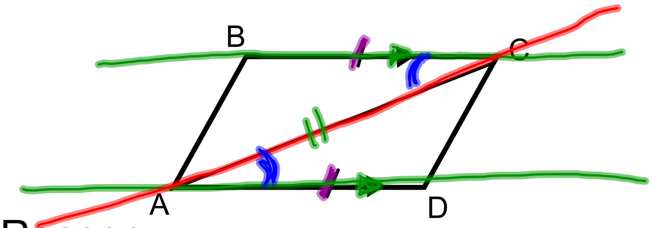
included ∠

If Side $\overline{RS} \cong \overline{UV}$,
Angle $\angle R \cong \angle U$, and
Side $\overline{RT} \cong \overline{UW}$,
then $\triangle RST \cong \triangle UVW$.

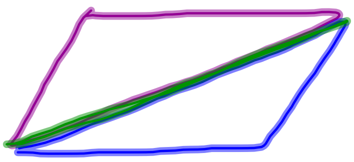


Given: $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

Prove: $\Delta ABC \cong \Delta CDA$

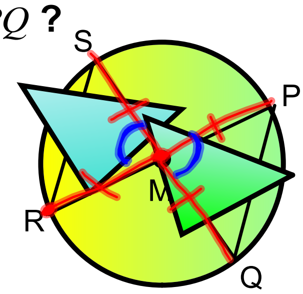


Statement	Reason
$\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$	Given
$\angle CAD \cong \angle ACB$	A.I. ∠s
$\overline{AC} \cong \overline{CA}$	Reflexive
$\Delta ABC \cong \Delta CDA$	S.A.S.



In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle.

What can you conclude about ΔMRS and ΔMPQ ?



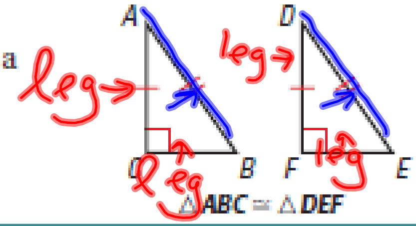
Statement	Reason
$\overline{QS}$, $\overline{RP}$ pass thru the ctr ⊙	Given
$\overline{RM}\cong\overline{QM}\cong\overline{PM}\cong\overline{SM}$	∀ pts on the ⊙ to the ctr are the same length.
$\angle SMR \cong \angle QMP$	V.A.T
$\Delta MRS \cong \Delta MPQ$	S.A.S

If a given angle is a RIGHT angle, ~~SSA can be~~ used to prove congruence.
If this is the case, it is called the Hypotenuse-Leg Congruence Theorem.
HL

THEOREM *For Your Notebook*

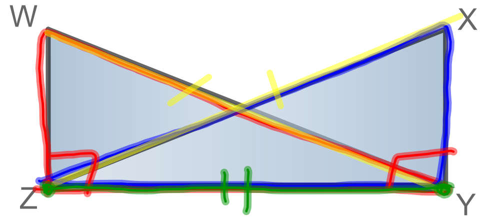
THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



Use HL to prove $\Delta WYZ \cong \Delta XZY$

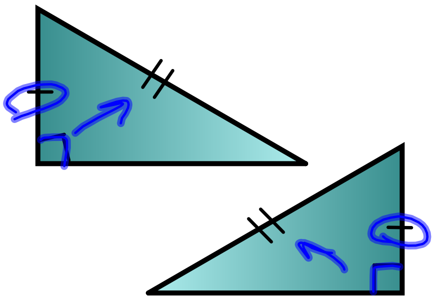
Given: $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$



Statement	Reason
H $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	Given
$\angle WZY$, $\angle XYZ$ are rt $\angle$s	def'n of $\perp$
$\angle WZY \cong \angle XYZ$	rt $\angle \cong$ Thm
leg $\overline{ZY} \cong \overline{YZ}$	Reflexive
$\Delta WYZ \cong \Delta XZY$	HL

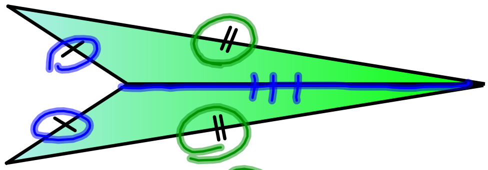
State whether or not the following pairs of triangles must be congruent.
If so, name the postulate that is used.

A



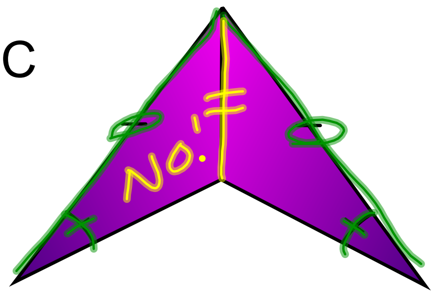
HL

B

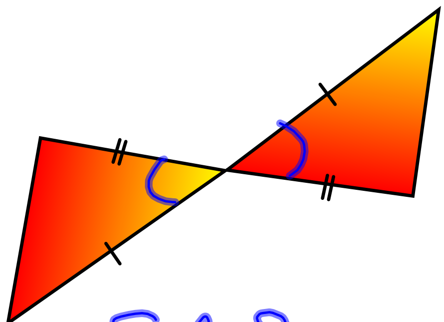


SSS

C



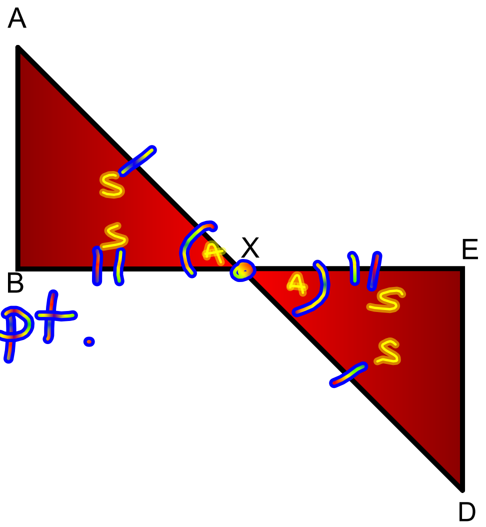
D



SAS

Given: X is the midpoint of $\overline{AD}$ and $\overline{BE}$

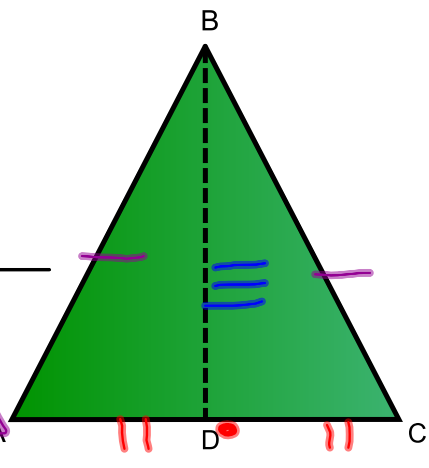
Prove: $\Delta AXB \cong \Delta DXE$



Statement	Reason
X is the midpoint of $\overline{AD}$ X is the midpoint of $\overline{BE}$	Given
$\overline{AX} \cong \overline{DX}$, $\overline{BX} \cong \overline{EX}$	def'n of midpt.
$\angle AXB \cong \angle DXE$	V.A.T.
$\Delta AXB \cong \Delta DXE$	SAS

Given: ΔABC is isosceles
D is the midpoint of $\overline{AC}$

Prove: $\Delta ABD \cong \Delta CBD$



Statement	Reason
ABC is isosceles D is the midpoint of $\overline{AC}$	Given
$\overline{AB} \cong \overline{CB}$	def'n of isos Δ
$\overline{AD} \cong \overline{CD}$	def'n of midpt.
$\overline{BD} \cong \overline{BD}$	Reflexive
$\Delta ABD \cong \Delta CBD$	SSS